Determination of the $\eta$-transition form factor in the $\gamma p \rightarrow p\eta \rightarrow p\gamma e^+e^-$ reaction

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Abstract

The Dalitz decay $\eta \rightarrow \gamma e^+e^-$ has been measured using the combined Crystal Ball and TAPS photon detector setup at the electron accelerator MAMI-C. Compared to the most recent transition form-factor measurement in the $e^+e^-$ channel, statistics have been improved by one order of magnitude. The $e^+e^-$ invariant mass distribution shows a deviation from the QED prediction for a point-like particle, which can be described by a form-factor. Using the usual monopole transition form-factor parameterization, $F(m^2) = (1 - m^2/\Lambda^2)^{-1}$, a value of $\Lambda^{-2} = (1.92 \pm 0.35(stat) \pm 0.13(syst))$ GeV$^{-2}$ has been determined. This value is in good agreement with a recent measurement of the $\eta$ Dalitz decay in the $\mu^+\mu^-$ channel and with recent form-factor calculations. An improved value of the branching ratio $BR(\eta \rightarrow \gamma e^+e^-) = (6.6 \pm 0.4(stat) \pm 0.4(syst)) \cdot 10^{-3}$ has been determined.

Keywords: meson decays, transition form factor

1. Introduction

Electromagnetic transition form factors are an important tool for studying the intrinsic structure of hadrons. For point-like particles the decay rates can be exactly calculated within Quantum Electrodynamics (QED). Information on the intrinsic structure is encoded in the multiplicative transition form factor. The decay of light pseudoscalar mesons $\pi^0, \eta,$ and $\eta'$ into a real and a virtual photon are of particular interest as they allow a test of the validity of the vector meson dominance (VMD) assumption where the coupling of a photon to a hadron is mediated by a virtual vector meson. Furthermore, the transition form factor can be studied for momentum transfers not accessible in scattering or annihilation experiments. New theoretical efforts aim at a systematic description of the dynamics of light pseudoscalar and vector mesons and their coupling to electromagnetism [1]. Here electromagnetic transition form factors of vector mesons [2] and pseudoscalar mesons [3] provide an excellent tool to test the predictions of the leading-order calculations and to stimulate calculations beyond leading order. These theoretical
activities have demonstrated the need for improved experimental data. An overview of earlier work in the field can be found in [4].

In this work the Dalitz decay of the $\eta$ meson $\eta \rightarrow \gamma \gamma \rightarrow \gamma e^+e^-$ has been investigated. The invariant mass distribution of the dilepton pairs is given by [4]

$$\frac{d\Gamma(\eta \rightarrow l^+l^-\gamma)}{dm\Gamma(\eta \rightarrow \gamma\gamma)} = \frac{4\alpha}{3\pi m} \cdot [1 - \frac{4m_l^2}{m^2}]^{\frac{1}{2}} \cdot [1 + \frac{2m_l^2}{m^2}] \cdot [1 - \frac{m^2}{m_\eta^2}]^3 \cdot |F_\eta(m^2)|^2,$$

$$= [QED] \cdot |F_\eta(m^2)|^2. \quad (1)$$

Here, $l$ stands for either $e$ or $\mu$, depending on the decay channel chosen; $m$ corresponds to the mass of the dilepton pair and $m_\eta$ to the mass of the $\eta$ meson, respectively. In the vector meson dominance assumption the form factor is usually parametrized by

$$F(m^2) = \frac{1}{1 - \frac{m^2}{\Lambda^2}}. \quad (2)$$

The only parameter in this one-pole approximation is the slope $b$ of the form factor for $m^2 = 0$, which is related to the effective virtual vector meson mass $\Lambda$ by

$$b = \left| \frac{dF}{dm^2} \right|_{m^2=0} = \Lambda^{-2}. \quad (3)$$

Measurements of this form factor have been reported by the SND collaboration [5] in the $\eta \rightarrow \gamma e^+e^-$ channel and by the Lepton-G experiment [6] in the $\eta \rightarrow \gamma \mu^+\mu^-$ decay mode. More recently a precision measurement was performed by the NA60 collaboration [7] in the $\mu^+\mu^-$ channel, but without detecting the photon. With a value of $\Lambda^{-2} = (1.6 \pm 2.0) \text{GeV}^{-2}$ the accuracy of the SND experiment was insufficient to establish a deviation from the QED prediction, which was the main motivation to repeat this measurement in the $\eta \rightarrow \gamma e^+e^-$ channel under improved conditions.

2. Experiment

The experiment was performed at the electron accelerator MAMI-C in Mainz using the combined Crystal Ball (CB) [8] and TAPS [9] detectors.
Energy-tagged photons were produced via the bremsstrahlung process in the Glasgow-Mainz tagging facility [10, 11] and impinged on a liquid hydrogen target. The quasi-monochromatic photon beam covered an energy range from 617 to 1402 MeV and had an intensity of \( 2 \times 10^5 \text{ (4 MeV)}^{-1} \text{s}^{-1} \) at 620 MeV. The bremsstrahlung photons, produced by the electrons in a 10 \( \mu \)m copper radiator, were collimated by a lead collimator that was 4 mm in diameter; the resulting diameter of the photon beam spot on the target was approximately 1 cm. The target itself had a diameter of 3 cm and a length of 4.76 cm and was located in the center of the Crystal Ball detector. The material budget around the target, including the Kapton cell and the 1 mm thick carbon-fiber beamline, amounted to 0.8% of a radiation length \( X_0 \), which was essential for suppressing the conversion of real photons into \( e^+e^- \) pairs.

By combining the Crystal Ball with the TAPS calorimeter in a forward wall configuration, photons and charged particles were registered over the full azimuthal angular range. Polar angles of \( 1^\circ - 20^\circ \) were subtended by the 384 BaF\(_2\) scintillator modules (12 \( X_0 \)) of TAPS and polar angles of \( 21^\circ - 160^\circ \) were covered by the 672 NaI scintillator modules (15.7 \( X_0 \)) of the CB. Charged particles were identified in plastic scintillators of 2 mm and 5 mm thickness mounted in front of the NaI and BaF\(_2\) scintillator modules, respectively. A more detailed description of the detector setup has recently been given in [12].

The data were collected during two running periods totalling 350 h. In addition, about 70 hours with a beam of twice the photon beam intensity were used for a measurement with an empty target. The trigger threshold for the total energy deposited in the Crystal Ball detector was 350 MeV; furthermore, a multiplicity trigger was used requiring two or more hits with energy depositions larger than 20 MeV in the TAPS or CB detectors (for details see [13]). In the data analysis clusters of at least 50 MeV are requested to suppress photon split-offs (see [14]).

3. Analysis Procedure

The reconstruction of a weak channel like \( \gamma p \rightarrow p\eta \rightarrow p\gamma e^+e^- \) among several strong background channels is a major experimental challenge. The identification of the final-state particles is hampered by the fact that the detector system does not provide a magnetic field for particle tracking. Only the energy and angle of the registered particles are available as well as the information whether they are charged or neutral. The sign of the charge
cannot be determined. Electrons and pions can be separated from protons by the energy loss in the plastic scintillators. A rough separation between electrons and pions is achieved by requesting for electron candidates a higher multiplicity of responding detector modules within a given cluster in the calorimeters. For pions corresponding multiplicities are typically 2-3.

Only events with at least one proton, one photon and two electron candidates were selected for further processing. A rare final state like in the $\eta \rightarrow \gamma e^+e^-$ Dalitz decay can only be identified among the many background channels by exploiting the complete kinematics of the $\gamma p \rightarrow p\eta \rightarrow p\gamma e^+e^-$ reaction in an exclusive analysis. Several cuts were applied to select the reaction of interest in the sequence given in Table 1.

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<td>2D-Cut, see Fig. 3</td>
</tr>
<tr>
<td>2</td>
<td>Momentum Balance Y vs. Energy Balance</td>
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<tr>
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<td>10</td>
<td>Angle $e^\pm\gamma$ [°]</td>
<td>50 to 175</td>
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Table 1: Applied cuts in the $\eta$-Dalitz analysis of simulated and real-data events.

The same cuts were applied to Monte Carlo simulations of possible background reactions. Typically 2-10 million events were simulated for the background channels, assuming phase space distributions, and 5 million events for the $\gamma p \rightarrow p\eta \rightarrow p\gamma e^+e^-$ reaction. For the latter the PLUTO event generator [15] was used. All together 67 cut settings were investigated in order to find a compromise between statistics in the channel of interest and background suppression. The rejection power of each cut and the remaining acceptance for the $\eta$ Dalitz decay are shown in Fig. 1 for the cut setting finally chosen. The dependence of the $p\gamma e^+e^-$ acceptance on the $m_{e^+e^-}$ mass is shown in Fig. 2. The fall off towards small invariant masses arises from cuts on the
cluster energies and opening angles and introduces systematic uncertainties estimated to be 5%. The mass-averaged acceptance is (2.0 ± 0.1)%.

Pions misidentified as electrons were rejected by cuts on the energy and momentum balance calculated from the entrance channel and all registered final-state particles (cuts 1, 2) as demonstrated by simulations shown in Fig. 3. An additional cut on the nucleon missing mass led to a suppression of $\pi^+\pi^-$ pairs relative to $e^+e^-$ pairs of $1 \cdot 10^{-7}$. $\eta \rightarrow 2\gamma$ decays with a subsequent conversion of one photon in the target or surrounding material were particularly suppressed by a cut on the opening angle of the $e^+e^-$ pair, as one would expect for real photon conversion (cut 8).

The events surviving all cuts listed in Table 1 are shown in Fig. 4 where the invariant $e^+e^-$ mass is plotted versus the $\gamma e^+e^-$ mass. On top of a smooth background distribution, the projection onto the x-axis exhibits a peak at the $\eta$ mass with $1345 \pm 59$ counts and a resolution of $\sigma = 14.1 \pm 0.4$ MeV consistent with simulations. A band near the $\pi^0$ mass of 135 MeV seen in the two-dimensional plot corresponds to $2\pi^0$ production events where
Figure 2: Acceptance for the $\gamma p \rightarrow p\eta \rightarrow p\gamma e^+e^-$ reaction as a function of the $e^+e^-$ invariant mass simulated with the PLUTO event generator [15] and the GEANT 4 code.

Figure 3: Simulation of the energy-momentum balance for the $\gamma p \rightarrow p\eta \rightarrow p\gamma e^+e^-$ reaction (left) and for the $\gamma p \rightarrow p\eta \rightarrow p\pi^+\pi^-\pi^0$ reaction (right). In the latter case the charged pions are assumed to be misidentified as electrons.
Figure 4: Distribution of reconstructed $e^+e^-\gamma$ candidate events after applying the cuts given in Table 1. The lower figure shows the projection onto the $e^+e^-\gamma$ mass axis.
two photons from a $\pi^0 \rightarrow \gamma\gamma$ decay were misidentified as electrons and one photon is not detected. The intensity of $\eta \rightarrow \gamma\gamma \rightarrow \gamma e^+ e^-$ as a function of the $e^+ e^-$ invariant mass $\text{Mass}_{e^+ e^-}$ was determined by fitting the $\gamma e^+ e^-$ mass distributions for different slices in $\text{Mass}_{e^+ e^-}$ as shown in Fig. 5. In the fits the peak position and the width were constrained to 540-560 MeV and 11-20 MeV, respectively. The fit errors increase with mass and reach a value of $\approx 50\%$ for the highest mass bins.

4. Results

Using Eq.1 for extracting the transition form factor from the data implies the determination of the $\eta \rightarrow e^+ e^- \gamma$ yield relative to the number of $\eta \rightarrow \gamma\gamma$ decays. An analysis of the latter decay mode, using the acceptance of 12.1% derived from Monte Carlo simulations for the $\gamma p \rightarrow p\eta \rightarrow p\gamma\gamma$ reaction, determined the number of $\eta \rightarrow \gamma\gamma$ decays to be $(4.01 \pm 0.18) \cdot 10^6$. Using the branching ratio $\text{BR}(\eta \rightarrow \gamma\gamma) = 39.3\%$ listed in the Review of Particle Physics [16] the total number of produced $\eta$ mesons corresponds to $(10.2 \pm 0.45) \cdot 10^6$. With the 1345 reconstructed $\eta \rightarrow \gamma e^+ e^-$ decays and the acceptance of $(2.0 \pm 0.1)\%$, a ratio of decay widths $\Gamma_{\eta \rightarrow \gamma e^+ e^-} / \Gamma_{\eta \rightarrow \gamma\gamma} = (1.68 \pm 0.10) \cdot 10^{-2}$ was deduced. This corresponds to a branching ratio $\text{BR}(\eta \rightarrow \gamma e^+ e^-) = (6.6 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})) \cdot 10^{-3}$ to be compared to the current PDG value.
of $(7.0 \pm 0.7) \cdot 10^{-3}$.

After correcting for the $e^+e^-$ mass-dependent acceptance of the $\gamma p \rightarrow p\eta \rightarrow p\gamma e^+e^-$ reaction (see Fig. 2) the invariant $e^+e^-$ mass distribution shown in Fig. 6 was obtained. The spectrum was fitted according to Eq. 1 with two parameters, the slope $b$ of the form factor and a normalization constant.

Dividing the data points in Fig. 6 by the QED prediction (see Eq.1) the $\eta$ transition form factor shown in Fig. 7 was deduced. The present data show an enormous improvement in the $e^+e^-$ channel compared to the previous work of the SND collaboration who reconstructed only 110 $\eta \rightarrow \gamma e^+e^-$ events. With 1345 reconstructed events the statistics could be improved by an order of magnitude. The statistics obtained by the NA60 collaboration are clearly superior but, due to choosing the $\mu^+\mu^-$ channel, their data points only start at dilepton masses above 200 MeV/c$^2$. The data points in the present analysis get closer to the photon point and reach invariant mass values as low as 40 MeV/c$^2$. Furthermore, due to the identification of all final-state particles, the $\eta$ meson could be fully reconstructed in contrast to
Figure 7: $\eta$-Dalitz transition form factor: The red circles are the data of this work (the black curve is the fit to the data). The magenta (full) squares show the result of the SND experiment [5]. The inverted (blue) triangles represent the result obtained by NA60 [7] in the $\mu^+\mu^-$ channel. The green (dashed) curve is a calculation performed by [3].
the NA60 analysis where the photon was not detected and the form factor was deduced by unfolding the $\mu^+\mu^-$ invariant mass spectrum, which also contained contributions from other particles.

From the fit, the slope of the $\eta$-meson transition form factor was determined as

$$b = \left| \frac{dF}{dm^2} \right|_{m^2=0} = \Lambda^{-2} = (1.92 \pm 0.35(stat) \pm 0.13(syst)) \text{ GeV}^{-2}, \quad (4)$$

corresponding to $\Lambda = (720 \pm 60(stat) \pm 50(syst)) \text{ MeV}$. The systematic uncertainty was estimated from a comparison of the results obtained in the 67 different cut settings. These parameters are more precise than the earlier SND result in the $e^+e^-$ channel of $\Lambda^{-2} = (1.6 \pm 2.0) \text{ GeV}^{-2}$ [5] and are consistent with the values reported in the $\mu^+\mu^-$ channel by Lepton-G: $\Lambda^{-2} = (1.9\pm0.4) \text{ GeV}^{-2}$ [6] and by NA60: $\Lambda^{-2} = 1.95\pm0.17(stat)\pm0.05(syst) \text{ GeV}^{-2}$ [7]. The value for the $\Lambda$ parameter is in excellent agreement with the predictions of the VMD model of $\Lambda = 730$-750 MeV, depending on the $\eta-\eta'$ mixing angle [4, 17]. The fit to the present data is very close to the calculation by Terschlüsen and Leupold [2, 3] who studied form factors in the decays of light mesons in a recently proposed scheme that treats pseudoscalar and vector mesons as active degrees of freedom [1]. This work calls for further improved experimental data.

5. Conclusion and Outlook

In this work the transition form factor of the $\eta$-Dalitz decay was measured in the $\eta \rightarrow e^+e^+\gamma$ channel with statistics improved by an order of magnitude compared to the most recent measurement in this channel [5]. The result is in good agreement with previous measurements from [6, 7] and with most recent theoretical calculations [2, 3]. After establishing that weak channels like the $\eta$ Dalitz decay can be identified in an exclusive analysis using a 4$\pi$ calorimeter without magnetic field, this experimental approach can be extended to the measurement of form factors of other pseudoscalar and vector mesons.

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